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Inverse Kinematics Analysis and Path Planning for 6DOF RSS Parallel Manipulator

Alena A. Yagur
ITMO University,
St. Petersburg, Russia
E-mail: ljolja1995@gmail.com

Alexey A. Belov
ITMO University,
St. Petersburg, Russia
V. A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
E-mail: a.a.belov@inbox.ru

***Abstract*—Spatial parallel manipulators have a number of practical applications due to their high reliability, accuracy, and performance. In this paper we deal with an inverse kinematic problem for six-degree-of-freedom (6DOF) parallel manipulator known as modified Stewart platform with rotative-spherical-spherical (RSS) structure. An effective analytic method for solving the inverse kinematic problem for given terminal state is proposed. This method is used for trajectory planning of 6DOF parallel manipulator. Numerical simulation is considered.**

***Index Terms*—path planning, manipulators, parallel robots**

I. Introduction

In the past years, parallel mechanisms have been case of study of many researchers due to their applications in industry research and machinery design. Currently, devices based on mechanisms of parallel kinematics are widely used as positioning devices, manipulators and micromanipulators, vibration stands, simulators, measuring systems, etc. The multidirectional closed kinematic loop of the mechanism leads to a decrease in the dimensions and masses of the moving links. Such devices apply the load like space trusses, which determine their increased accuracy and carrying capacity.

Parallel manipulators have several advantages: better load capacity, high accuracy of positioning of the working element, higher rigidity of the system, high speeds and accelerations of the working element, high degree of unification of mechatronic knots. Spatial precision positioning devices are often based on hexapods or tripods. To overcome the limitations of the serial manipulator many researchers proposed spatial parallel manipulator including Stewart platform. Many kinds of literature are available in the field of spatial parallel manipulator [1], [2].

The hexapod is one form of parallel manipulator that is used increasingly in manufacturing, inspection and research. The ultimate hexapod would provide large motions for massive payloads in up to six degrees of freedom with high accuracy, resolution and repeatability.

Some of the works related to the derivation of the inverse kinematics, forward kinematics, workspace analysis and singularity analysis of the spatial platform are listed in the references [4]–[7]. A number of related papers is dedicated to the inverse kinematics problem for parallel manipulator structures [8]–[10].

Singular point and behavior of the parallel platforms at their neighborhood are investigated in papers [11], [12]. In [11] a method for analysis of numerical location of singular points of in-parallel actuated manipulators has been proposed, and the neighborhood of the singular points has been determined based on the kinematic and static characteristics of in-parallel manipulators. The operating area of in-parallel actuated manipulators has been obtained using transmission index. For details, see [11], [12].

In this paper we propose a simple solution of the inverse kinematics problem for hexapod with rotative-spherical-spherical (RSS) structure. This solution allows to find operating area zone of hexapod and to solve path planning problem. The novelty of proposed method consists of trigonometric function approach. The solution of inverse kinematics is obtained in analytic form. In addition, a problem of multiple choice of angular orientation of actuators is removed.

The rest paper is organized as follows. In Section II description of the platform, necessary notations, and problem statement are considered. Section III establishes the main results: inverse kinematics analysis, operating area zone estimation, and path planning. Conclusive remarks are given in Section IV.

II. Platform description and problem statement

The mechanism of the parallel structure with rotary drives (modified Stewart platform) has 6 degrees of freedom and consists of six kinematic chains containing one rotative and two spherical pairs (see Fig. 1). Construction pattern of the manipulator is presented in Fig. 2.

Denote input rotative pairs of the section as A_i where $i = \overline{1,6}$ is a number of rotative pairs. Output spherical pair of the section are denoted as B_i. Medium spherical pairs are denoted as C_i. Input and output links are assumed to be ideal discs. Now we assign local coordinate systems with input and output sections. Moreover, center of the local coordinate system is assumed to be in the center of the disc. Axes $o_j x_j$

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Fig. 1. 3D-model of the parallel manipulator with 6 rotary drives.

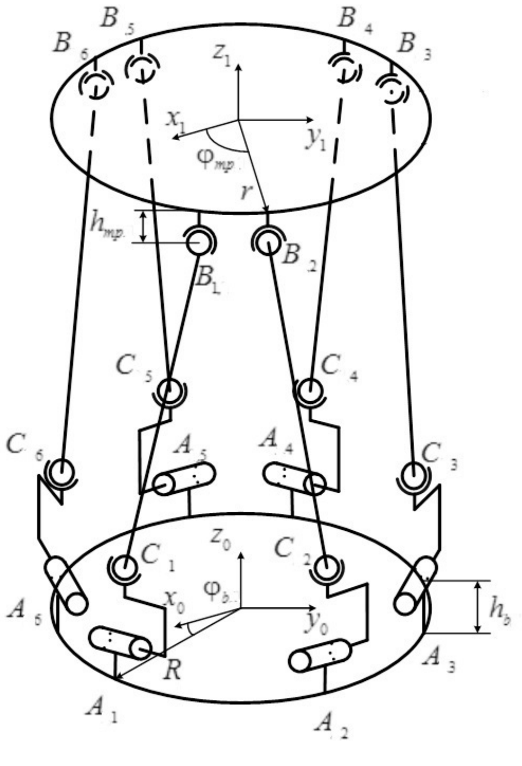

Fig. 2. Construction pattern of parallel structure manipulator with 6 rotary drives.

and $o_j y_j$, $j = 0, 1$ lie on the surface of lower and upper discs respectively. Axes $o_j z_j$ are orthogonal to these surfaces. Let input and output pairs be located on the vertexes of regular hexagons with radiuses R and r respectively. Lengths of the segments $A_i C_i$ and $C_i B_i$ are denoted l_1 and l_2 respectively.

Joint coordinates are defined through angular coordinate of the first joint φ_b and angular coordinate of the moving platform φ_{mp}. We will use the angles between the coupling link connecting the input kinematic pair A_i with the medium spherical pair C_i, and the normal to the plane of the platform base as the generalized coordinates of this mechanism. Generalized coordinates are denoted as θ_i, $i = \overline{1, 6}$.

Inverse kinematic analysis problem for parallel structure manipulator can be formulated in the following way.

Problem: for a given position of the center point and angle position of the output section find generalized coordinates of the platform, i.e. angle positions θ_i.

III. Main Results

A. Inverse Kinematic Analysis

In order to solve this problem, we write uniform coordinates of the joints in fixed coordinate system as follows:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

$$\begin{aligned}
a_{1,1} &= R\cos\varphi_b, & a_{2,1} &= R\sin\varphi_b, \\
a_{1,2} &= R\cos\left(\tfrac{2\pi}{3} - \varphi_b\right), & a_{2,2} &= R\sin\left(\tfrac{2\pi}{3} - \varphi_b\right), \\
a_{1,3} &= R\cos\left(\tfrac{2\pi}{3} + \varphi_b\right), & a_{2,3} &= R\sin\left(\tfrac{2\pi}{3} + \varphi_b\right), \\
a_{1,4} &= R\cos\left(\tfrac{4\pi}{3} - \varphi_b\right), & a_{2,4} &= R\sin\left(\tfrac{4\pi}{3} - \varphi_b\right), \\
a_{1,5} &= R\cos\left(\tfrac{4\pi}{3} + \varphi_b\right), & a_{2,5} &= R\sin\left(\tfrac{4\pi}{3} + \varphi_b\right), \\
a_{1,6} &= R\cos(-\varphi_b), & a_{2,6} &= R\sin\varphi_b,
\end{aligned}$$

and $a_{3,1} = a_{3,2} = a_{3,3} = a_{3,4} = a_{3,5} = a_{3,6} = h_b$.

Uniform coordinates of the moving platform in moving coordinates are equal to

$$\tilde{B} = \begin{bmatrix} \tilde{b}_{1,1} & \tilde{b}_{1,2} & \tilde{b}_{1,3} & \tilde{b}_{1,4} \\ \tilde{b}_{2,1} & \tilde{b}_{2,2} & \tilde{b}_{2,3} & \tilde{b}_{2,4} \\ \tilde{b}_{3,1} & \tilde{b}_{3,2} & \tilde{b}_{3,3} & \tilde{b}_{3,4} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{2}$$

where

$$\begin{aligned}
\tilde{b}_{1,1} &= R\cos\varphi_{mp}, & \tilde{b}_{2,1} &= R\sin\varphi_{mp}, \\
\tilde{b}_{1,2} &= R\cos\left(\tfrac{2\pi}{3} - \varphi_{mp}\right), & \tilde{b}_{2,2} &= R\sin\left(\tfrac{2\pi}{3} - \varphi_{mp}\right), \\
\tilde{b}_{1,3} &= R\cos\left(\tfrac{2\pi}{3} + \varphi_{mp}\right), & \tilde{b}_{2,3} &= R\sin\left(\tfrac{2\pi}{3} + \varphi_{mp}\right), \\
\tilde{b}_{1,4} &= R\cos\left(\tfrac{4\pi}{3} - \varphi_{mp}\right), & \tilde{b}_{2,4} &= R\sin\left(\tfrac{4\pi}{3} - \varphi_{mp}\right), \\
\tilde{b}_{1,5} &= R\cos\left(\tfrac{4\pi}{3} + \varphi_{mp}\right), & \tilde{b}_{2,5} &= R\sin\left(\tfrac{4\pi}{3} + \varphi_{mp}\right), \\
\tilde{b}_{1,6} &= R\cos -\varphi_{mp}, & \tilde{b}_{2,6} &= R\sin\varphi_{mp},
\end{aligned}$$

and $\tilde{b}_{3,1} = \tilde{b}_{3,2} = \tilde{b}_{3,3} = \tilde{b}_{3,4} = \tilde{b}_{3,5} = \tilde{b}_{3,6} = -h_b$.

Uniform transformation matrix describing the transition between moving and fixed coordinate systems is considered to be known, and equal to

$$T = \begin{bmatrix} d_{1,1} & d_{1,2} & d_{1,3} & x_0 \\ d_{2,1} & d_{2,2} & d_{2,3} & y_0 \\ d_{3,1} & d_{3,2} & d_{3,3} & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

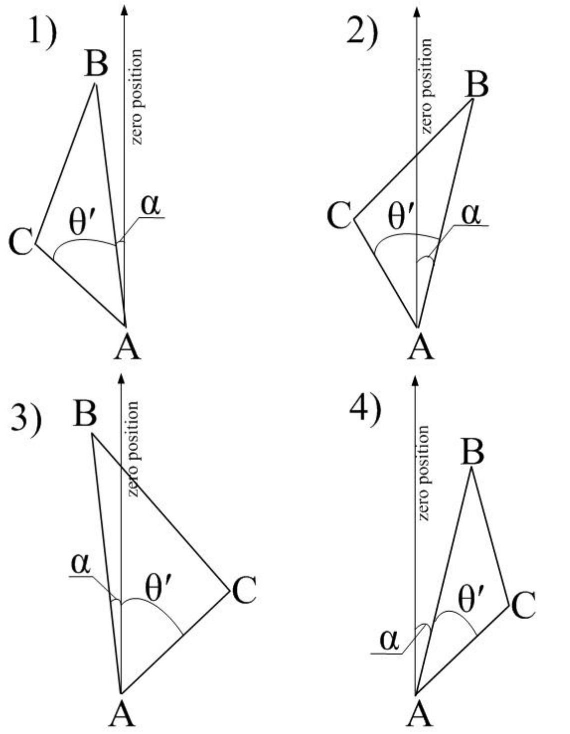

Fig. 3. Possible directions of rotation of the i-th kinematic chain.

where $d_{i,j},\ i,j = \overline{1,3}$ are direction cosines of unit normal vector, and $x_0,\ y_0,\ z_0$ are coordinates of the displacement vector.

Taking into account (2) and (3), we get coordinates of joints of the moving platform as

$$B = T\tilde{B}. \tag{4}$$

To solve inverse kinematic problem, we consider i-th kinematic chain of the platform. Let the positive direction of rotation be clockwise direction. Link A_iC_i rotates in both clockwise and counterclockwise directions. This leads to multiple solutions of the problem. Possible ways of rotation are depicted in Fig. 3. Variants of rotation in clockwise direction corresponds to pictures 3) and 4) in Fig. 3. Variants of rotation in counterclockwise direction corresponds to pictures 1) and 2) in Fig. 3. It is easy to see, that generalized coordinates θ_i are defined as

$$\theta_i = \theta_i^{'} + \alpha_i \tag{5}$$

or

$$\theta_i = \theta_i^{'} - \alpha_i \tag{6}$$

for the cases 1), 4) and 2), 3) respectively.

Angle $\theta_i^{'}$ of the triangle $A_iC_iB_i$ can be found from cosine theorem as follows

$$\theta_i^{'} = \operatorname{acos}\left(\frac{l_2^2 - l_1^2 - \|A_iB_i\|^2}{-2l_1\|A_iB_i\|}\right) \tag{7}$$

where $\|A_iB_i\|^2 = (b_{i,1}-a_{i,1})^2+(b_{i,2}-a_{i,2})^2+(b_{i,3}-a_{i,3})^2$.

TABLE I
HEXAPOD PARAMETERS

Description	Value	Unit
Base radius, R	170	mm
Moving platform (MP) radius, r	160	mm
Angle coordinate of first base joint, ϕ_n	$\pi/6$	rad
Angle coordinate of first MP joint, ϕ_{mp}	$\pi/18$	rad
Distance from base joints to rotation axis, h_n	45	mm
Distance from MP joints to rotation axis, h_{mp}	15	mm
Length of the link, l_1	68	mm
Length of the link, l_2	168	mm

Angle α_i are defined as

$$\alpha_i = \operatorname{atan}\left(\frac{b_{i,2} - \tilde{b}_{i,2}}{b_{i,3}}\right). \tag{8}$$

Here $b_{i,j}$ are elements of matrix B from (4).

B. Operating area zone analysis

We consider a 6DOF parallel manipulator with the following parameters (see Table I).

Solution of the inverse kinematic problem (7)–(8) can be applied in construction of operating area zone. In order to avoid multiple solutions of the inverse kinematic problem, we consider that angle coordinates of moving links A_iC_i for $i = 1,2,3$ are defined by (5) when $y_0 \geqslant 0$ and by (6) otherwise. Angle coordinates of moving links A_iC_i for $i = 4,5,6$ are defined by (6) when $y_0 \geqslant 0$ and by (5) otherwise. Rotation angles of rotary drives are limited by $-90...90$ degrees. For the simplicity, we suppose that angle position of the moving platform is equal to zero (i.e. the moving platform is parallel to the base section). We propose the following algorithm to construct operating area zone:

1) Define the space area $\mathcal{O} \in \mathbb{R}^3$ of possible position of the moving platform center and step of grid h.
2) For each point $O_k(x,y,z) \in \mathcal{O}$ solve inverse kinematic problem (7)–(8).
3) Check if solution $\theta_i,\ i = \overline{1,6}$ satisfy the following conditions:
   C1: θ_i ia real value,
   C2: $-90 \leqslant \theta_i \leqslant 90$.
4) If conditions C1–C2 are satisfied, then O_k belongs to the operating area zone
5) Repeat steps 2)–4) for all points of the area $\mathcal{O}$.

We used the following parameters $x_O \in [-130;\ 130]$, $y_O \in [-130;\ 130]$, $z_O \in [210;\ 290]$, and grid step $h = 0.5$. Operating area zone built using above mentioned algorithm is depicted in Fig. 4. Axe z corresponds to the height of the hexapod in millimeters. Contour curves of the upper bound and lower bound of operating area zone are depicted in Figs. 5 and 6 respectively. at the right of the contour areas there is a legend of hexapod height.

C. Path planning

Results, obtained above, can be successfully used in path planning problem. In this case the problem is to find a set of

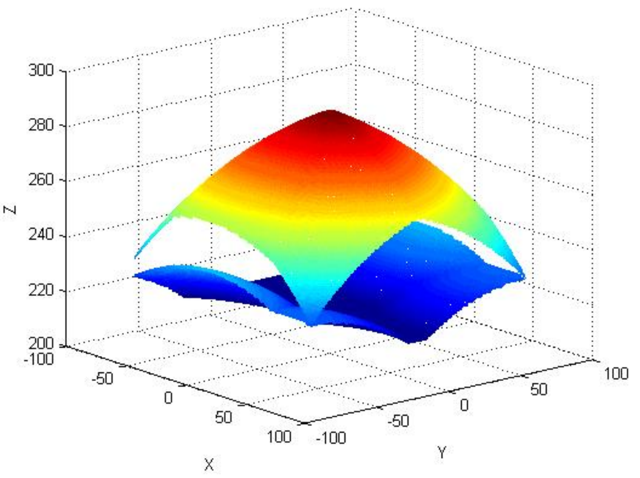

Fig. 4. Operating area zone.

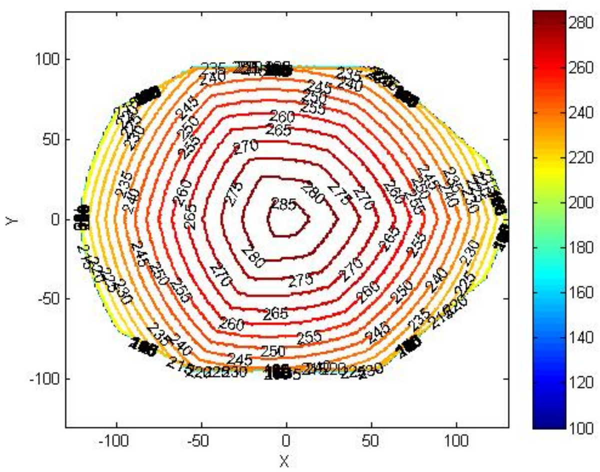

Fig. 5. Contour curves of upper bound of the operating area zone.

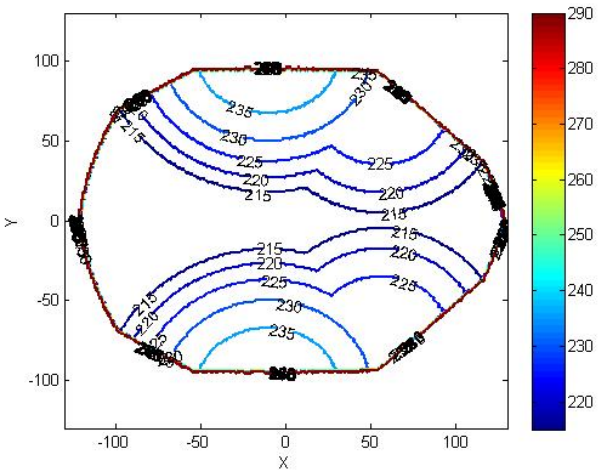

Fig. 6. Contour curves of lower bound of the operating area zone.

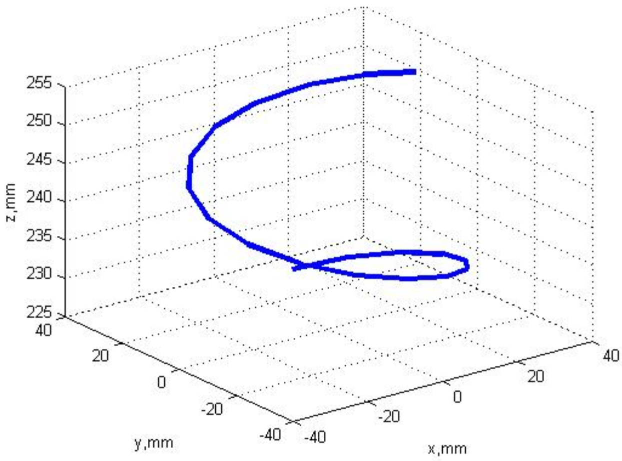

Fig. 7. Reference trajectory.

angle positions of the rotary drives that allows to follow given spatial curve given as a set of points. Clearly, given trajectory should lie inside operating area.

To illustrate effectiveness of the proposed inverse kinematic problem solution in trajectory planning problem, we consider the following reference trajectory, describing spatial spiral curve represented in Fig. 7:

$$\begin{cases} x = 30 \sin 0.4s, \\ y = 30 \cos 0.4s, \\ z = 1.5s + 225 \end{cases} \tag{9}$$

where $s = \overline{1, 20}$ is a parameter.

Since trajectory (9) lies inside operating area zone, the solution (7)–(8) satisfy constraints C1–C2. The solution to the path following problem (9) is presented in Fig. 8.

IV. Conclusion

The paper proposes a method for solving the inverse kinematic problem for the modified Stewart platform with rotational kinematic pairs (six degrees of freedom) using matrix algebra. The advantage of this solution is that it can be generalized to other configurations of the Stewart platform. It is only necessary to determine the geometric parameters of the robot, which give us a solution to the problem taking into account the desired position and orientation.

This method was successfully applied to the estimation of the operating area zone of the 6DOF parallel manipulator and path planning problem. Obtained results can be applied in industrial and human robotics.

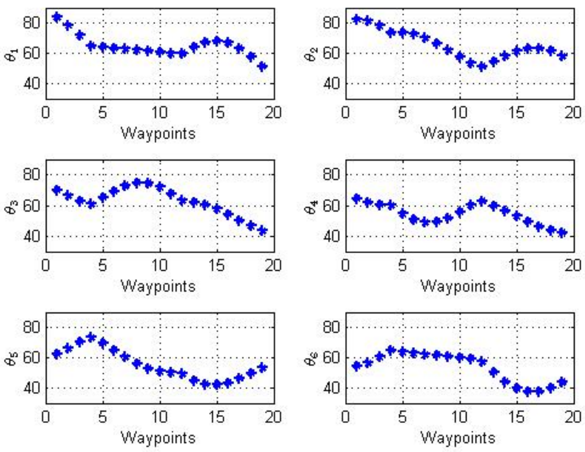

Fig. 8. Solution to the path following problem (9).

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